

## Research Article

# Hybrid Adaptive Bionic Fuzzy Control Method

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Received 10 April 2017; Revised 11 July 2017; Accepted 2 August 2017; Published 18 October 2017

Academic Editor: R. Aguilar-López

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The bionic fuzzy system embeds the biologically active adaptive strategy into the traditional T-S fuzzy system, which increases the active adaptability. On the basis of the researches, an identification model is added to the system, and a hybrid bionic adaptive fuzzy control method is proposed in this paper, which makes the system have biological adaptability and strong anti-interference ability. The adaptive law contains two items: the first one is the general term for adjusting system parameters by using the current state and the second one is a compensation item for the adjustment of system parameters based on the development trend. *Lyapunov* synthesis method is used to analyze the stability and convergence of system. The design method of fuzzy controller, adaptive laws, and parameter constraints are given. Finally, the effectiveness of the method is verified by simulation of inverted pendulum model.

## 1. Introduction

Due to the high complexity of dynamic environment and system structure, the research of nonlinear systems is difficult to achieve accurate description. At the same time, higher requirements on the stability and effectiveness of the controlled system needed to meet certain performance index. By combining the adaptive control strategy and fuzzy control method, the stability and anti-interference of the system can be improved due to the active adaptability of the organism [1, 2]. The niche of individual organism is introduced into the design of fuzzy systems, and adaptive behavior and individual feedback are used to establish the fuzzy control model based on ecological niche [3]. The design method of genetic algorithm and double nested fuzzy control method are given to make optimized controllable niche structure. A tracking feedback control method based on optimum niche is proposed, which is applied in the intelligent greenhouse system to realize the fuzzy control design for optimal ecological system [4]. In [5, 6], the self-organization and self-learning of niche are added to the fuzzy system, and a new fuzzy control method based on niche model is given. At the same time, the universal approximation of system was demonstrated, which achieved good results by using the system function as approximation. In [7], the T-S adaptive fuzzy control method is established by using the niche proximity

with normal distribution characteristics as the consequent of fuzzy rules. The research [8] gives  $H^\infty$  tracking performance designs in both indirect and direct adaptive fuzzy systems.

On the basis of the study, in this paper, combining the identification model [9, 10] with bionic fuzzy system and mixing the model error with tracking error, the hybrid bionic fuzzy control method is built. Stability and convergence analyzed by *Lyapunov* synthesis method [11–13], adaptive law, and constraints of the system parameters are given. The two items of bionic adaptive law, respectively, represent the current state and development trend of systems, which makes the system have good active adaptability. Finally, the strong anti-interference and better stability of highly complexity nonlinear system are verified by the simulation [14–18].

## 2. Basic Theory of Fuzzy System

*2.1. Hybrid Adaptive Fuzzy System.* Consider the following nonlinear systems:

$$\begin{aligned} \dot{x}^{(n)} &= f(x, \dot{x}, \dots, x^{(n-1)}) + g(x, \dot{x}, \dots, x^{(n-1)})u \\ y &= x. \end{aligned} \quad (1)$$

$f(x)$  and  $g(x)$  are continuous functions;  $u, y \in R$  are the input and output of the system.  $x = (x_1, x_2, \dots, x_n)^T = (x, \dot{x}, \dots, x^{(n-1)})^T \in R^n$  is obtained by measuring the state

vector. In order to control the system, when  $x$  belongs to a controllable interval  $U_c \subset R^n$ ,  $g(x) \neq 0$  might as well set  $g(x) > 0$ .

The ideal controllers in the form

$$u^* = \frac{1}{g(x)} [-f(x) + y_m^{(n)} + k^T e] \quad (2)$$

set  $e = y_m - y = y_m - x$ ,  $e = (e, \dot{e}, \dots, e^{(n-1)})^T$ ,  $k = (k_n, k_{n-1}, \dots, k_1)^T$  and make the real part of all the roots of polynomial  $s^n + k_1 s^{n-1} + \dots + k_n$  negative. When  $t \rightarrow \infty$ ,  $e(t) \rightarrow 0$ . That is, the output of the system converges  $y$  asymptotically to the ideal output  $y_m$ .

A hybrid adaptive controller is constructed as follows [12]:

$$u = \alpha u_c + (1 - \alpha) u_D + u_s + u_I. \quad (3)$$

$u_c$  is equivalent controller,  $u_D$  is output controller,  $u_s$  is supervisor controller,  $u_I$  is adaptive compensation controller, and  $\alpha \in [0, 1]$  is weighted factor.

## 2.2. Bionic Fuzzy System

**2.2.1. Niche Ecostate-Ecorole Theory Function.** Considering the ecosystem with  $n$  biological unit, the size of the niche of the  $k$  biological unit is defined as

$$N_k = \frac{s_k + C_k p_k}{\sum_{j=1}^n s_j + C_j p_j}. \quad (4)$$

$s_j$  indicates the state of the  $j$  biological unit,  $C_j$  is the dimensional conversion coefficient, and  $p_j$  indicates the rate

of change of the  $j$  biological unit,  $j = 1, 2, \dots, n$ . If the biological individual has a good state and good development trend, then the niche of the biological individual is relatively large,  $N_k \in [0, 1]$ . When the niche  $N_k$  of the  $k$  biological unit is greater, it indicates the greater niche role relatively in the biological unit in the system in study.

**2.2.2. Bionic Fuzzy System Rules.** Considering system (1),  $f(x)$  and  $g(x)$  are unknown functions, and the fuzzy system is constructed by a series of "if-then" fuzzy rules:

$R_f^i$ : if  $x_1$  is  $A_1^i$  and  $x_2$  is  $A_2^i \dots$  and  $x_n$  is  $A_n^i$ , then

$$f^i(x) = N_k^i, \quad i = 1, 2, \dots, M_f.$$

$R_g^i$ : if  $x_1$  is  $B_1^i$  and  $x_2$  is  $B_2^i \dots$  and  $x_n$  is  $B_n^i$ , then

$$g^i(x) = N_k^i, \quad i = 1, 2, \dots, M_g.$$

$k = 1, 2, \dots, n$ ,  $R^i$  represents  $i$  fuzzy rule, and  $A_k^i$  and  $B_k^i$  are the fuzzy sets of state vector  $x_k$ , determined by the Ecostate  $s_k^i$  and the Ecorole  $p_k^i$  of the  $k$  biological unit.  $N_k^i = (s_k^i + C_k p_k^i) / (\sum_{m=1}^n s_m^i + C_m p_m^i)$  is the output of the  $i$  rule.

Using the center-average defuzzifier, product inference engine, singleton fuzzifier, and Gauss membership function, Adaptive fuzzy system based on biological adaptive strategy is as follows:

$$\hat{f}(x | \theta_{1f}, \theta_{2f}) = \frac{\sum_{i=1}^M ((s_k^i + C_k p_k^i) / (\sum_{j=1}^n s_j^i + C_j p_j^i)) \prod_{j=1}^n \exp[-((x_j - \bar{x}_j^i) / \delta_j^i)^2]}{\sum_{i=1}^M \prod_{j=1}^n \exp[-((x_j - \bar{x}_j^i) / \delta_j^i)^2]} = \theta_{1f}^T \xi(x) + C_k \theta_{2f}^T \xi(x), \quad (5)$$

where

$$\begin{aligned} \theta_{1f} &= (\theta_{1f}^1, \theta_{1f}^2, \dots, \theta_{1f}^M)^T, \\ \theta_{1f}^i &= \frac{s_k}{\sum_{j=1}^n (s_j^i + C_j p_j^i)}, \\ \theta_{2f} &= (\theta_{2f}^1, \theta_{2f}^2, \dots, \theta_{2f}^M)^T, \\ \theta_{2f}^i &= \frac{p_k^i}{\sum_{m=1}^n (s_m^i + C_m p_m^i)}, \\ \xi(x) &= (\xi^1(x), \xi^2(x), \dots, \xi^M(x))^T, \end{aligned} \quad (6)$$

$$\xi^i(x) = \frac{\prod_{j=1}^n \exp[-((x_j - \bar{x}_j^i) / \delta_j^i)^2]}{\sum_{i=1}^M \prod_{j=1}^n \exp[-((x_j - \bar{x}_j^i) / \delta_j^i)^2]}.$$

Similarly,

$$\hat{g}(x | \theta_{1g}, \theta_{2g}) = \theta_{1g}^T \eta(x) + C_k \theta_{2g}^T \eta(x), \quad (7)$$

$$u_D(x | \theta_{1D}, \theta_{2D}) = \theta_{1D}^T \varphi(x) + C_k \theta_{2D}^T \varphi(x), \quad (8)$$

where  $\theta_{1f}$ ,  $\theta_{1g}$ ,  $\theta_{1D}$  are on behalf of the current state of the biological individuals and  $\theta_{2f}$ ,  $\theta_{2g}$ ,  $\theta_{2D}$  represent development trend of the biological individuals in the future.

## 3. Bionic Hybrid Controller

A nonlinear system is introduced in parallel with system (1):

$$\dot{\hat{x}}_1 = x_2,$$

$$\dot{\hat{x}}_2 = x_3$$

$$\begin{aligned} & \vdots \\ \dot{\hat{x}}_n &= \hat{f}(x | \theta_{1f}, \theta_{2f}) + \hat{g}(x | \theta_{1g}, \theta_{2g})u. \end{aligned} \quad (9)$$

Define model error as

$$\varepsilon = \dot{\hat{x}}_n(t) - \dot{x}_n(t). \quad (10)$$

From system 3 and system (1), it can be obtained that

$$\begin{aligned} \varepsilon &= [\hat{f}(x | \theta_{1f}, \theta_{2f}) - f(x)] \\ &+ [\hat{g}(x | \theta_{1g}, \theta_{2g}) - g(x)]u. \end{aligned} \quad (11)$$

The model error of the original system and the identification model combined as

$$\begin{aligned} \varepsilon &= \phi_{1f}^T \xi(x) + C\phi_{2f}^T \xi(x) + \phi_{1g}^T \eta(x)u \\ &+ \phi_{2g}^T C\eta(x)u + \omega, \\ \phi_{1f} &= \theta_{1f} - \theta_{1f}^*, \\ \phi_{2f} &= \theta_{2f} - \theta_{2f}^*, \\ \phi_{1g} &= \theta_{1g} - \theta_{1g}^*, \\ \phi_{2g} &= \theta_{2g} - \theta_{2g}^* \end{aligned} \quad (12)$$

using  $\hat{f}(x | \theta_{1f}, \theta_{2f})$ ,  $\hat{g}(x | \theta_{1g}, \theta_{2g})$ ,  $u_D(x | \theta_{1D}, \theta_{2D})$  replace  $\hat{f}(x)$ ,  $\hat{g}(x)$ , and  $u_D(x)$ ; the minimum approximation error is

$$\begin{aligned} \omega &= \alpha [\hat{f}(x | \theta_{1f}^*, \theta_{2f}^*) - f(x) \\ &+ (\hat{g}(x | \theta_{1g}^*, \theta_{2g}^*) - g(x))u_c] + (1 - \alpha)g(x)(u^* \\ &- u_D(x | \theta_{1D}, \theta_{2D})). \end{aligned} \quad (13)$$

Tracking error can be written as

$$\begin{aligned} \dot{e} &= \Lambda_c e + b_c \{ \alpha (\hat{f}(x | \theta_{1f}, \theta_{2f}) - f(x) \\ &+ [\hat{g}(x | \theta_{1g}, \theta_{2g}) - g(x)]u_c) \} + b_c (1 - \alpha)g(x) \\ &\cdot (u^* - u_D(x | \theta_{1D}, \theta_{2D})) - b_c g(x)(u_s + u_I). \end{aligned} \quad (14)$$

Following design  $u_s$  satisfies  $\dot{V} \leq 0$ , and design supervision controller as

$$\begin{aligned} u_s &= \left(\frac{V}{\bar{V}}\right)^p \text{sgn}(e^T P b_c) \left[ \frac{\alpha}{g_1(x)} |\hat{f}(x | \theta_{1f}, \theta_{2f})| \right. \\ &+ f_1(x) (|\hat{g}(x | \theta_{1g}, \theta_{2g})| + g_2(x) |u_c|) \\ &+ (f_1(x) + |y_m^{(n)}| + k^T e) \left. \right] + \left(\frac{V}{\bar{V}}\right)^p \text{sgn}(e^T P b_c) (1 \\ &- \alpha) [|u_D(x | \theta_{1D}, \theta_{2D})| \\ &+ g_1(x) (f_1(x) + |y_m^{(n)}| + |k^T e|) + g_2(x) |u_I|]. \end{aligned} \quad (15)$$

Choose the compensation controller as

$$u_I = c_0 \text{sgn}(e^T P b_c). \quad (16)$$

$c_0$  is positive number; then the fuzzy controller of the closed loop system is

$$\begin{aligned} u &= \alpha \frac{1}{\hat{g}(x | \theta_{1g}, \theta_{2g})} [-\hat{f}(x | \theta_{1f}, \theta_{2f}) + |y_m^{(n)}| \\ &+ |k_0^T e|] + (1 - \alpha)u_D(x | \theta_{1D}, \theta_{2D}) + u_s + u_I. \end{aligned} \quad (17)$$

If  $V \leq \bar{V}$ , the supervisory controller is 0; if the system tends to separate,  $V \geq \bar{V}$ , using  $u_s$  to ensure  $V \leq \bar{V}$ .

#### 4. The Design of Adaptive Laws

Adjusting optimal parameters of hybrid fuzzy system based on Bionic  $\theta_{1f}^*$ ,  $\theta_{2f}^*$ ,  $\theta_{1g}^*$ ,  $\theta_{2g}^*$ ,  $\theta_{1D}^*$ ,  $\theta_{2D}^*$

$$\begin{aligned} & (\theta_{1f}^*, \theta_{2f}^*) \\ &= \arg \min_{\theta_{1f} \in \Omega_{1f}, \theta_{2f} \in \Omega_{2f}} \left[ \sup_{x \in u_I} |f(x | \theta_{1f}, \theta_{2f}) - f(x)| \right], \\ & (\theta_{1g}^*, \theta_{2g}^*) \\ &= \arg \min_{\theta_{1g} \in \Omega_{1g}, \theta_{2g} \in \Omega_{2g}} \left[ \sup_{x \in u_I} |g(x | \theta_{1g}, \theta_{2g}) - g(x)| \right], \\ & (\theta_{1D}^*, \theta_{2D}^*) \\ &= \arg \min_{\theta_{1D} \in \Omega_{1D}, \theta_{2D} \in \Omega_{2D}} \left[ \sup_{x \in u_I} |u(x | \theta_{1D}, \theta_{2D}) - u(x)| \right] \\ \Omega_{1f} &= \{ \theta_{1f}^l : \|\theta_{1f}^l\| \leq M_{1f} \}, \\ \Omega_{2f} &= \{ \theta_{2f}^l : \|\theta_{2f}^l\| \leq M_{2f} \}, \\ \Omega_{1g} &= \{ \theta_{1g}^l : \delta \leq \|\theta_{1g}^l\| \leq M_{1g} \}, \\ \Omega_{2g} &= \{ \theta_{2g}^l : \delta \leq \|\theta_{2g}^l\| \leq M_{2g} \}, \\ \Omega_{1D} &= \{ \theta_{1D}^l : \|\theta_{1D}^l\| \leq M_{1D} \}, \\ \Omega_{2D} &= \{ \theta_{2D}^l : \|\theta_{2D}^l\| \leq M_{2D} \}. \end{aligned} \quad (18)$$

$\Omega_{1f}$ ,  $\Omega_{2f}$ ,  $\Omega_{1g}$ ,  $\Omega_{2g}$ ,  $\Omega_{1D}$ ,  $\Omega_{2D}$  are, respectively, the constraint set of  $\theta_{1f}$ ,  $\theta_{2f}$ ,  $\theta_{1g}$ ,  $\theta_{2g}$ ,  $\theta_{1D}$ ,  $\theta_{2D}$  and  $M_{1f}$ ,  $M_{2f}$ ,  $M_{1g}$ ,  $M_{2g}$ ,  $M_{1D}$ ,  $M_{2D}$ ,  $\delta$  are given constant.

Consider the followed Lyapunov function:

$$\begin{aligned} V &= \frac{1}{2} e^T P e + \frac{\alpha}{2\gamma_{1f}} (\theta_{1f} - \theta_{1f}^*)^T (\theta_{1f} - \theta_{1f}^*) \\ &+ \frac{\alpha}{2\gamma_{2f}} (\theta_{2f} - \theta_{2f}^*)^T (\theta_{2f} - \theta_{2f}^*) \end{aligned}$$

$$\begin{aligned}
& + \frac{\alpha}{2\gamma_{1g}} (\theta_{1g} - \theta_{1g}^*)^T (\theta_{1g} - \theta_{1g}^*) \\
& + \frac{\alpha}{2\gamma_{2g}} (\theta_{2g} - \theta_{2g}^*)^T (\theta_{2g} - \theta_{2g}^*) \\
& + \frac{(1-\alpha)}{2\gamma_{1D}} (\theta_{1D} - \theta_{1D}^*)^T (\theta_{1D} - \theta_{1D}^*) \\
& + \frac{\alpha}{2\gamma_{2D}} (\theta_{2D} - \theta_{2D}^*)^T (\theta_{2D} - \theta_{2D}^*).
\end{aligned} \tag{20}$$

$P$  is a positive definite matrix and satisfies the *Lyapunov* equation, and  $Q$  is an arbitrary positive definite matrix.

The bionic adaptive laws of the identification system are as follows:

$$\begin{aligned}
\dot{\theta}_{1f} &= -\gamma_{1f} [\gamma\varepsilon + e^T P b_c] \xi(x), \\
\dot{\theta}_{2f} &= -\gamma_{2f} [\gamma\varepsilon + e^T P b_c C] \xi(x), \\
\dot{\theta}_{1g} &= -\gamma_{1g} [\gamma\varepsilon + e^T P b_c] \eta(x) u,
\end{aligned}$$

$$\begin{aligned}
\dot{\theta}_{2g} &= -\gamma_{2g} [\gamma\varepsilon + e^T P b_c C] \eta(x) u, \\
\dot{\theta}_{1D} &= -\gamma_{1D} [\gamma\varepsilon + g(x) e^T P b_c] \varphi(x), \\
\dot{\theta}_{2D} &= -\gamma_{2D} [\gamma\varepsilon + g(x) e^T P b_c C] \varphi(x),
\end{aligned} \tag{21}$$

where  $\theta_{1f}$ ,  $\theta_{1g}$ ,  $\theta_{1D}$  represent the current state and  $\theta_{2f}$ ,  $\theta_{2g}$ ,  $\theta_{2D}$  represent the development trend of systems, which makes the systems have good active adaptability.

## 5. Analysis of System Stability and Convergence

It will affect the stability when the system is disturbed. In this paper, a hybrid adaptive fuzzy controller is designed, and the adaptive law is given. The following analysis is conducted for the performance of fuzzy controller. To ensure  $\theta_{1f} \in \Omega_{1f}$ ,  $\theta_{2f} \in \Omega_{2f}$ ,  $\theta_{1g} \in \Omega_{1g}$ ,  $\theta_{2g} \in \Omega_{2g}$ ,  $\theta_{1D} \in \Omega_{1D}$ ,  $\theta_{2D} \in \Omega_{2D}$ , parametric projection method is designed to satisfy the parameter vector.

In terms of  $\hat{f}(x | \theta_{1f}, \theta_{2f})$ ,

$$\begin{aligned}
\dot{\theta}_{1f} &= \begin{cases} -\gamma_{1f} [\gamma\varepsilon + e^T P b_c] \xi(x) & \text{when } (\|\theta_{1f}\| < M_{1f} \text{ or } \|\theta_{1f}\| = M_{1f}, e^T P b_c \xi(x) \theta_{1f}^T \geq 0) \\ P \{-\gamma_{1f} [\gamma\varepsilon + e^T P b_c] \xi(x)\} & \text{when } (\|\theta_{1f}\| = M_{1f}, e^T P b_c \xi(x) \theta_{1f}^T < 0) \end{cases} \\
\dot{\theta}_{2f} &= \begin{cases} -\gamma_{2f} [\gamma\varepsilon + e^T P b_c] \xi(x) & \text{when } (\|\theta_{2f}\| < M_{2f} \text{ or } \|\theta_{2f}\| = M_{2f}, e^T P b_c \xi(x) \theta_{2f}^T \geq 0) \\ P \{-\gamma_{2f} [\gamma\varepsilon + e^T P b_c] \xi(x)\} & \text{when } (\|\theta_{2f}\| = M_{2f}, e^T P b_c \xi(x) \theta_{2f}^T < 0) \end{cases}
\end{aligned} \tag{22}$$

which satisfies the projection operator

$$\begin{aligned}
& P \{-\gamma_{1f} [\gamma\varepsilon + e^T P b_c] \xi(x)\} \\
& = -\gamma_{1f} [\gamma\varepsilon + e^T P b_c] \xi(x) \\
& \quad + \gamma_{1f} [\gamma\varepsilon + e^T P b_c] \frac{\theta_{1f}^T \theta_{1f} \xi(x)}{|\theta_{1f}|^2}.
\end{aligned} \tag{23}$$

In terms of  $\hat{g}(x | \theta_{1g}, \theta_{2g})$ , if one element of  $\theta_{1g}$ ,  $\theta_{2g}$  satisfies  $\theta_{1g}^i = \delta$ ,  $\theta_{2g}^i = \delta$ , then

$$\begin{aligned}
\dot{\theta}_{1g}^i &= \begin{cases} -\gamma_{1g} [\gamma\varepsilon + e^T P b_c] \eta^i(x) u & (\gamma\varepsilon + e^T P b_c) \eta^i(x) u < 0 \\ 0 & (\gamma\varepsilon + e^T P b_c) \eta^i(x) u \geq 0 \end{cases} \\
\dot{\theta}_{2g}^i &= \begin{cases} -\gamma_{2g} [\gamma\varepsilon + e^T P b_c C] \eta^i(x) u & (\gamma\varepsilon + e^T P b_c C) \eta^i(x) u \leq 0 \\ 0 & (\gamma\varepsilon + e^T P b_c C) \eta^i(x) u > 0. \end{cases}
\end{aligned} \tag{24}$$

Otherwise,

$$\begin{aligned}
\dot{\theta}_{1g} &= \begin{cases} -\gamma_{1g} [\gamma\varepsilon + e^T P b_c] \eta(x) & \text{when } (\|\theta_{1g}\| < M_{1g} \text{ or } \|\theta_{1g}\| = M_{1g}, e^T P b_c \eta(x) \theta_{1g}^T \geq 0) \\ P \{-\gamma_{1g} [\gamma\varepsilon + e^T P b_c] \eta(x)\} & \text{when } (\|\theta_{1g}\| = M_{1g}, e^T P b_c \eta(x) \theta_{1g}^T < 0) \end{cases} \\
\dot{\theta}_{2g} &= \begin{cases} -\gamma_{2g} [\gamma\varepsilon + e^T P b_c C] \eta(x) & \text{when } (\|\theta_{2g}\| < M_{2g} \text{ or } \|\theta_{2g}\| = M_{2g}, e^T P b_c C \eta(x) \theta_{2g}^T \geq 0) \\ P \{-\gamma_{2g} [\gamma\varepsilon + e^T P b_c C] \eta(x)\} & \text{when } (\|\theta_{2g}\| = M_{2g}, e^T P b_c C \eta(x) \theta_{2g}^T < 0) \end{cases}
\end{aligned} \tag{25}$$

which satisfies the projection operator

$$P \{-\gamma_{1g} [\gamma \varepsilon + e^T P b_c] \eta(x)\} \\ = -\gamma_{1g} [\gamma \varepsilon + e^T P b_c] \eta(x)$$

$$+ \gamma_{1g} [\gamma \varepsilon + e^T P b_c] \frac{\theta_{1g}^T \theta_{1g} \eta(x)}{|\theta_{1g}|^2}. \quad (26)$$

In terms of  $u_D(x | \theta_{1D}, \theta_{2D})$ ,

$$\dot{\theta}_{1D} = \begin{cases} -\gamma_{1D} [\gamma \varepsilon + e^T P b_c g(x)] \varphi(x) & \text{when } (\|\theta_{1D}\| < M_{1D} \text{ or } \|\theta_{1D}\| = M_{1D}, e^T P b_c g(x) \varphi(x) \theta_{1D}^T \geq 0) \\ P \{-\gamma_{1D} [\gamma \varepsilon + e^T P b_c g(x)] \varphi(x)\} & \text{when } (\|\theta_{1D}\| = M_{1D} \text{ or } e^T P b_c g(x) \varphi(x) \theta_{1D}^T < 0) \end{cases} \quad (27)$$

$$\dot{\theta}_{2D} = \begin{cases} -\gamma_{2D} [\gamma \varepsilon + e^T P b_c g(x)] \varphi(x) & \text{when } (\|\theta_{2D}\| < M_{2D} \text{ or } \|\theta_{2D}\| = M_{2D}, e^T P b_c g(x) \varphi(x) \theta_{2D}^T \geq 0) \\ P \{-\gamma_{2D} [\gamma \varepsilon + e^T P b_c g(x)] \varphi(x)\} & \text{when } (\|\theta_{2D}\| = M_{2D}, e^T P b_c g(x) \varphi(x) \theta_{2D}^T < 0) \end{cases}$$

which satisfies the projection operator

$$P \{-\gamma_{1D} [\gamma \varepsilon + e^T P b_c] \varphi(x)\} \\ = -\gamma_{1D} [\gamma \varepsilon + e^T P b_c] \varphi(x) \quad (28)$$

$$+ \gamma_{1D} [\gamma \varepsilon + e^T P b_c g(x)] \frac{\theta_{1D}^T \theta_{1D} \varphi(x)}{|\theta_{1D}|^2}.$$

**Theorem 1.** Set the constraints set of  $\Omega_{1f}, \Omega_{2f}, \Omega_{1g}, \Omega_{2g}, \Omega_{1D}, \Omega_{2D}$  which are given by (19); if the initial value of the parameter satisfies  $\theta_{1f}(0) \in \Omega_{1f}, \theta_{2f}(0) \in \Omega_{2f}, \theta_{1g}(0) \in \Omega_{1g}, \theta_{2g}(0) \in \Omega_{2g}, \theta_{1D}(0) \in \Omega_{1D}, \theta_{2D}(0) \in \Omega_{2D}$ , to any  $t \geq 0$ , adaptive laws can ensure  $\theta_{1f}(t) \in \Omega_{1f}, \theta_{2f}(t) \in \Omega_{2f}, \theta_{1g}(t) \in \Omega_{1g}, \theta_{2g}(t) \in \Omega_{2g}, \theta_{1D}(t) \in \Omega_{1D}, \theta_{2D}(t) \in \Omega_{2D}$ .

*Proof.* Set  $V_{1f} = (1/2)\theta_{1f}^T \theta_{1f}$ , if formula (24) satisfies the first condition, when  $|\theta_{1f}| = M_{1f}, |\theta_{1f}| \leq M_{1f}$ . Otherwise,  $\dot{V}_{1f} = -\gamma_{1f}(\gamma \varepsilon + e^T P b_c \theta_{1f}^T) \xi(x) \leq 0$ , and  $|\theta_{1f}| \leq M_{1f}$ ; if formula (24) satisfies the second condition, then  $|\theta_{1f}| = M_{1f}$ , and  $\dot{V}_{1f} = -\gamma_{1f}[\gamma \varepsilon + e^T P b_c] \theta_{1f}^T \xi(x) + \gamma_{1f}[\gamma \varepsilon + e^T P b_c](\theta_{1f}^T \theta_{1f} \xi(x) / |\theta_{1f}|^2) \theta_{1f}^T \xi(x) = 0$ ; therefore, in this case,  $|\theta_{1f}| \leq M_{1f}$ , due to the initial conditions  $\theta_{1f}(0) \leq M_{1f}$ , to any  $t \geq 0, \theta_{1f}(t) \leq M_{1f}$ . Similarly, it can be proved that  $\theta_{2f}(t) \leq M_{2f}, \theta_{1g}(t) \leq M_{1g}, \theta_{2g}(t) \leq M_{2g}, \theta_{1D}(t) \leq M_{1D}$ , and  $\theta_{2D}(t) \leq M_{2D}$ .

It can be obtained by formula (24) that if  $\theta_{1g}^l = \delta, \dot{\theta}_{1g}^l \geq 0$ , then there is  $\theta_{1g}^l \geq \delta, |\theta_{1g}| \geq \delta$ . Similarly,  $\theta_{2g} \geq \delta$ .  $\square$

**Theorem 2.** Hybrid adaptive fuzzy systems form (1), where  $f(x)$  and  $g(x)$  are all unknown functions, and the equivalent controller  $u_c$ , the output controller  $u_D$ , the supervisory controller  $u_s$ , and the adaptive compensation controller  $u_t$  are, respectively, designed according to formulas (3), (8), (15), and (16). The adaptive law is given by (21), and adaptive fuzzy control system must have the following characteristics:

(1) All parameters and state variables are bounded:

$$\|\theta_{1f}^l\| \leq M_{1f}, \\ \|\theta_{2f}^l\| \leq M_{2f}, \\ \|\theta_{1g}^l\| \leq M_{1g}, \\ \|\theta_{2g}^l\| \leq M_{2g}, \|\theta_{1D}^l\| \leq M_{1D}, \\ \|\theta_{2D}^l\| \leq M_{2D}, \\ |x(t)| \leq M_x \quad (29)$$

and  $M_{1f}, M_{2f}, M_{1g}, M_{2g}, M_{1D}, M_{2D}, M_x$  are given positive constant.

(2) The boundary of tracking error and the model error is given by the minimum approximation error set  $\phi = (\phi_f + \phi_g u)^T \xi(x)$ ; then

$$\int_0^t |e(\tau)|^2 d\tau + \int_0^t |\phi(\tau)|^2 d\tau \leq a + b \int_0^t |\omega(\tau)|^2 d\tau, \quad \forall t \geq 0, \quad (30)$$

$$\int_0^t |e(\tau)|^2 d\tau + \int_0^t |\varepsilon(\tau)|^2 d\tau \leq a + b \int_0^t |\omega(\tau)|^2 d\tau, \quad \forall t \geq 0.$$

(3) If square of  $\omega$  is integrable, it means that  $\int_0^\infty |\omega(\tau)|^2 dt < \infty$ ; then  $\lim_{t \rightarrow \infty} |e(t)| = 0, \lim_{t \rightarrow \infty} |\varepsilon(t)| = 0$ .

*Proof.*  $\|\theta_{1f}^l\| \leq M_{1f}, \|\theta_{2f}^l\| \leq M_{2f}, \|\theta_{1g}^l\| \leq M_{1g}, \|\theta_{2g}^l\| \leq M_{2g}, \|\theta_{1D}^l\| \leq M_{1D}$ , and  $\|\theta_{2D}^l\| \leq M_{2D}$  have been proved in Theorem 1, and the following is to prove that  $|x| \leq M_x$ .

While designing the controller  $u_s$ , set  $V < \bar{V}$ ,  $\bar{V}$  is a given constant, set  $\lambda_{P_{\min}}$  is the minimum eigenvalue of  $P$ , and due to  $V = (1/2)e^T P e$ , it can be obtained that

$$V^{1/2} \geq \left( \frac{\lambda_{P_{\min}}}{2} \right)^{1/2} |e| \geq \left( \frac{\lambda_{P_{\min}}}{2} \right)^{1/2} (|x| - |y_m|). \quad (31)$$

Therefore,  $V \leq \bar{V}$  equals  $|x| \leq |y_m| + (2\bar{V}/\lambda_{P_{\min}})^{1/2}$ , and to satisfy  $|x| \leq M_x$ , it can choose

$$\bar{V} = \frac{\lambda_{P_{\min}}}{2} \left( M_x - \sup_{t \geq 0} |y_m| \right)^2. \quad (32)$$

□

*Proof.* Combine the adaptive laws (21), (22), (24), (25), and (26) into the following *Lyapunov* function:

$$\begin{aligned} \dot{V} = & -\frac{1}{2}e^T Q e + e^T P b_c \omega \\ & + \frac{\alpha}{\gamma_{1f}} \phi_{1f}^T [\dot{\theta}_{1f} + \gamma_{1f} e^T P b_c \xi(x)] \\ & + \frac{\alpha}{\gamma_{2f}} \phi_{2f}^T [\dot{\theta}_{2f} + \gamma_{2f} e^T P b_c C \xi(x)] \\ & + \frac{\alpha}{\gamma_{1g}} \phi_{1g}^T [\dot{\theta}_{1g} + \gamma_{1g} e^T P b_c \eta(x) u] \\ & + \frac{\alpha}{\gamma_{2g}} \phi_{2g}^T [\dot{\theta}_{2g} + \gamma_{2g} e^T P b_c C \eta(x) u] \\ & + \frac{1-\alpha}{\gamma_{1D}} \phi_{1D}^T [\dot{\theta}_{1D} + \gamma_{1D} e^T P b_c g(x) \varphi(x)] \\ & + \frac{1-\alpha}{\gamma_{2D}} \phi_{2D}^T [\dot{\theta}_{2D} + \gamma_{2D} e^T P b_c g(x) C \varphi(x)]. \end{aligned} \quad (33)$$

Then

$$\begin{aligned} \dot{V} = & -\frac{1}{2}e^T Q e + e^T P b_c \omega + \phi_{1f}^T [-\gamma \varepsilon \xi(x)] \\ & + \phi_{2f}^T [-\gamma \varepsilon C \xi(x)] + \sum_P \phi_{1g}^P [e^T P b_c \eta^P(x) u] \\ & + \sum_P \phi_{1g}^P [-\gamma \varepsilon \eta^P(x) u] \\ & + \sum_q \phi_{2g}^q [e^T P b_c C \eta^q(x) u] \\ & + \sum_q \phi_{2g}^q [-\gamma \varepsilon C \eta^q(x) u] + \phi_{1D}^T [-\gamma \varepsilon \varphi(x)] \\ & + \phi_{2D}^T [-\gamma \varepsilon C \varphi(x)] \end{aligned} \quad (34)$$

due to that,

$$\begin{aligned} & \sum_P \phi_{1g}^P [-\gamma \varepsilon \eta^P(x) u] \\ & + \sum_q \phi_{1g}^q [-\gamma \varepsilon \eta^q(x) u] = \sum_{l=1}^M \phi_{1g}^l [-\gamma \varepsilon \eta^l(x) u], \\ & \sum_P \phi_{2g}^P [-\gamma \varepsilon C \eta^P(x) u] \\ & + \sum_q \phi_{2g}^q [-\gamma \varepsilon C \eta^q(x) u] = \sum_{l=1}^M \phi_{2g}^l [-\gamma \varepsilon C \eta^l(x) u], \end{aligned} \quad (35)$$

$$\begin{aligned} & \phi_{1f}^T [-\gamma \varepsilon \xi(x)] + \phi_{1g}^T [-\gamma \varepsilon \eta(x) u] + \phi_{2f}^T [-\gamma \varepsilon C \xi(x)] \\ & + \phi_{2g}^T [-\gamma \varepsilon C \eta(x) u] = -\gamma \phi^2 - \lambda \phi \omega, \end{aligned}$$

and there is

$$\begin{aligned} \phi = & \phi_{1f}^T \xi(x) + C \phi_{2f}^T \xi(x) + \phi_{1g}^T \eta(x) u + \phi_{2g}^T C \eta(x) u \\ = & \varepsilon - \omega. \end{aligned} \quad (36)$$

Set

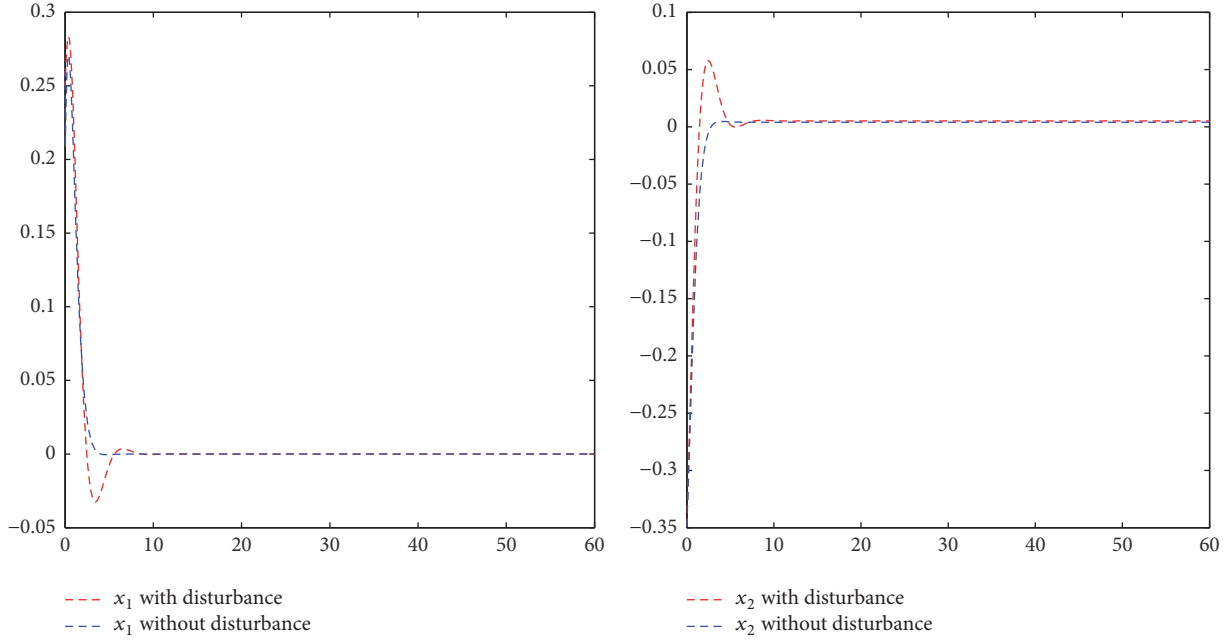
$$\begin{aligned} \tilde{e} &= \begin{bmatrix} e \\ \phi \end{bmatrix}, \\ \tilde{Q} &= \begin{bmatrix} Q & 0 \\ 0 & 2\gamma \end{bmatrix}, \\ \tilde{P}_n &= \begin{bmatrix} P b_c \\ -\gamma \end{bmatrix} \end{aligned} \quad (37)$$

so it can be obtained that

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2}e^T Q e + (e^T P b_c - \gamma \phi) \omega - \gamma \phi^2 \\ = & -\frac{1}{2} \tilde{e}^T \tilde{Q} \tilde{e} + \tilde{e}^T \tilde{P}_n \omega \\ \leq & -\frac{1}{2} \lambda_{\tilde{Q}_{\min}} |\tilde{e}|^2 + \frac{1}{2} |\tilde{e}|^2 + \frac{1}{2} |\tilde{P}_n \omega|^2 \\ = & -\frac{\lambda_{\tilde{Q}_{\min}} - 1}{2} |\tilde{e}|^2 + \frac{1}{2} |\tilde{P}_n \omega|^2 \end{aligned} \quad (38)$$

to any  $t \geq 0$ ,

$$\begin{aligned} \int_0^t |\tilde{e}(\tau)|^2 d\tau \leq & \frac{2}{\lambda_{\tilde{Q}_{\min}} - 1} [V(0) - V(t)] \\ & + \frac{|\tilde{P}_n|^2}{\lambda_{\tilde{Q}_{\min}} - 1} \int_0^t |\omega(\tau)|^2 d\tau \\ \leq & \frac{2}{\lambda_{\tilde{Q}_{\min}} - 1} [V(0)] \\ & + \frac{|\tilde{P}_n|^2}{\lambda_{\tilde{Q}_{\min}} - 1} \int_0^t |\omega(\tau)|^2 d\tau \end{aligned} \quad (39)$$


 FIGURE 1:  $x_1$  and  $x_2$  with internal disturbance.

and because of  $|\tilde{e}(t)|^2 = |e(t)|^2 + [\varepsilon(t)]^2$ ,  $|\tilde{P}_n|^2 = |Pb_c|^2 + |\gamma|^2$ , it can be proved by

$$\begin{aligned}
 a &= \frac{2V(0)}{(\min\{\lambda_{Q_{\min}}, 2\gamma\} - 1)}, \\
 b &= \frac{|Pb_c|^2 + \gamma^2}{(\min\{\lambda_{Q_{\min}}, 2\gamma\} - 1)}
 \end{aligned} \quad (40)$$

noting that  $\{\lambda_{Q_{\min}}, 2\gamma\} - 1 > 0$ .  $\square$

*Proof.* If  $\omega \in L_2$ , we can obtain  $e \in L_2$  by Theory 2, because the variables are bounded; then  $\dot{e} \in L_\infty$ , by Barbalat lemma (if  $e \in L_2 \cap L_\infty$  and  $\dot{e} \in L_\infty$ , then  $\lim_{t \rightarrow \infty} |e(t)| = 0$ ); we can get  $\lim_{t \rightarrow \infty} |e(t)| = 0$ .  $\square$

## 6. Simulation

*Example 1.* The inverted pendulum problem is studied by the hybrid adaptive fuzzy control method:

$$\begin{aligned}
 \dot{x}_1 &= x_2, \\
 \dot{x}_2 &= \frac{g \sin x_1 - mlx_2^2 \sin x_1 / (m_c + m)}{l(4/3 - m \cos^2 x_1 / (m_c + m))} \\
 &\quad + \frac{\cos x_1 / (m_c + m)}{l(4/3 - m \cos^2 x_1 / (m_c + m))} u
 \end{aligned} \quad (41)$$

and  $g = 9.8 \text{ m/s}^2$ ,  $m_c = 1 \text{ kg}$ ,  $m = 0.1 \text{ kg}$ ,  $l = 0.5 \text{ m}$ . Choosing  $K^T = (k_1 \ k_2) = (1 \ 2)$ ,  $C_k = 1$ ,  $Q = \text{diag}(10 \ 10)$ , by solving equation  $\Lambda_c^T P = P\Lambda_c - Q$ , then  $P = \begin{pmatrix} 15 & 5 \\ 5 & 5 \end{pmatrix}$ .

The membership function is

$$\begin{aligned}
 \mu_{M_1^1}(x_1) &= \exp\left[-\left(\frac{x_1 + (\pi/6)}{\pi/10}\right)^2\right], \\
 \mu_{M_1^2}(x_1) &= \exp\left[-\left(\frac{x_1}{\pi/10}\right)^2\right], \\
 \mu_{M_1^3}(x_1) &= \exp\left[-\left(\frac{x_1 - (\pi/6)}{\pi/10}\right)^2\right], \\
 \mu_{M_2^1}(x_2) &= \exp\left[-\left(\frac{x_2 + (\pi/6)}{\pi/10}\right)^2\right], \\
 \mu_{M_2^2}(x_2) &= \exp\left[-\left(\frac{x_2}{\pi/10}\right)^2\right], \\
 \mu_{M_2^3}(x_2) &= \exp\left[-\left(\frac{x_2 - (\pi/6)}{\pi/10}\right)^2\right].
 \end{aligned} \quad (42)$$

Set the initial conditions  $x(0) = (-\pi/9, \pi/12)^T$ , the original system, and control system with internal disturbance (system state input  $x$  is substituted by  $(1 + \delta_1)x$ ,  $\delta_1 = 0.5$ ) and external disturbance ( $\delta_2 = 0.01$ ).

From Figures 1–3, when the system is disturbed, the hybrid bionic adaptive controller proposed in this paper makes the system have more anti-interference and good convergence. In addition, this paper makes a comparison when the output trajectory has both internal and external disturbances; it can be seen that the fuzzy system has achieved good results. From Figure 4(a), when  $C_k = 1$ , the system has good convergence and biological adaptability than system controlled by traditional method. From Figure 4(b),

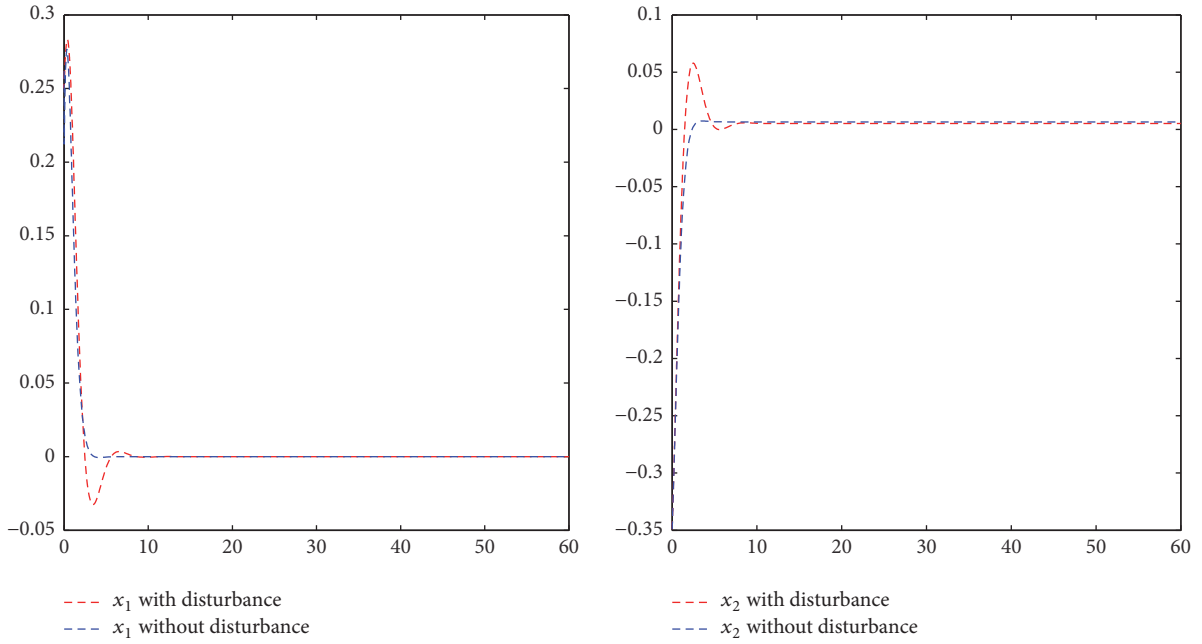


FIGURE 2:  $x_1$  and  $x_2$  with external disturbance.

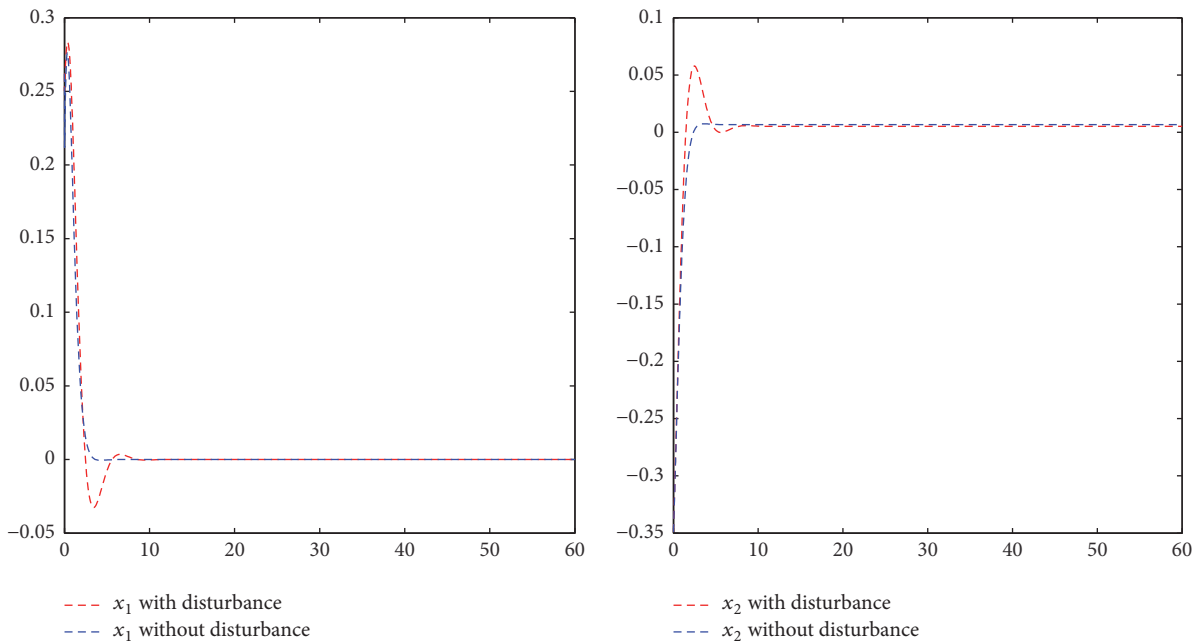


FIGURE 3:  $x_1$  and  $x_2$  with both internal and external disturbances.

by comparing the output trajectories, it can be obtained that the bionic hybrid fuzzy control has better result.

*Example 2.* Consider the following uncertain system:

$$\begin{aligned} \dot{\omega} &= -\omega + x_1^2 + x_2^2, \\ \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -x_1 - 2x_2 + \Delta f(x, v, t) + \Delta h(\omega) + u, \end{aligned} \tag{43}$$

where  $\omega \in R$  is the unmodelled dynamics of the system.  $\Delta h(\omega)$  is the related uncertain item of the unmodelled dynamics  $\omega$ .

Set

$$\begin{aligned} \Delta h(\omega) &= 2\omega, \\ \Delta f(x, v, t) &= \frac{b_1 + b_2 \cos(2t) x_1^2 x_2^2}{a_1 + a_2 \sin(t) + a_3 x_1^2 + a_4 x_2^2}. \end{aligned} \tag{44}$$



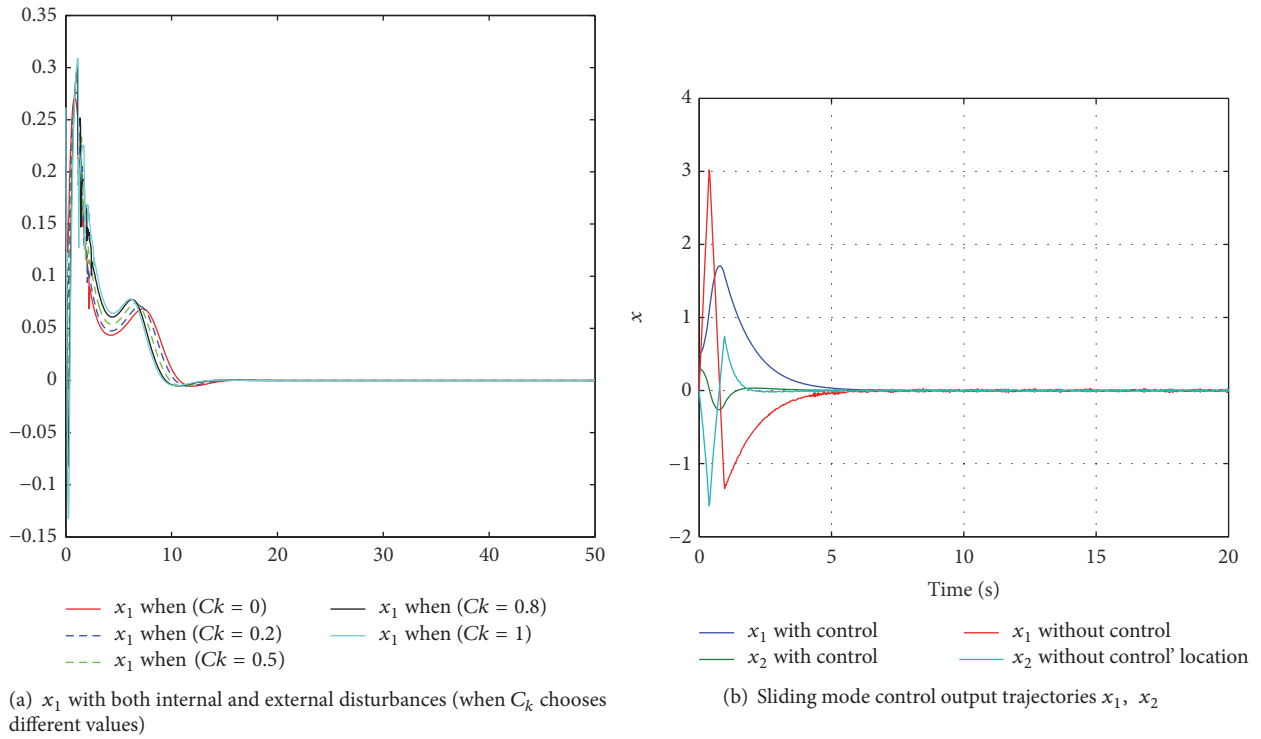


FIGURE 4

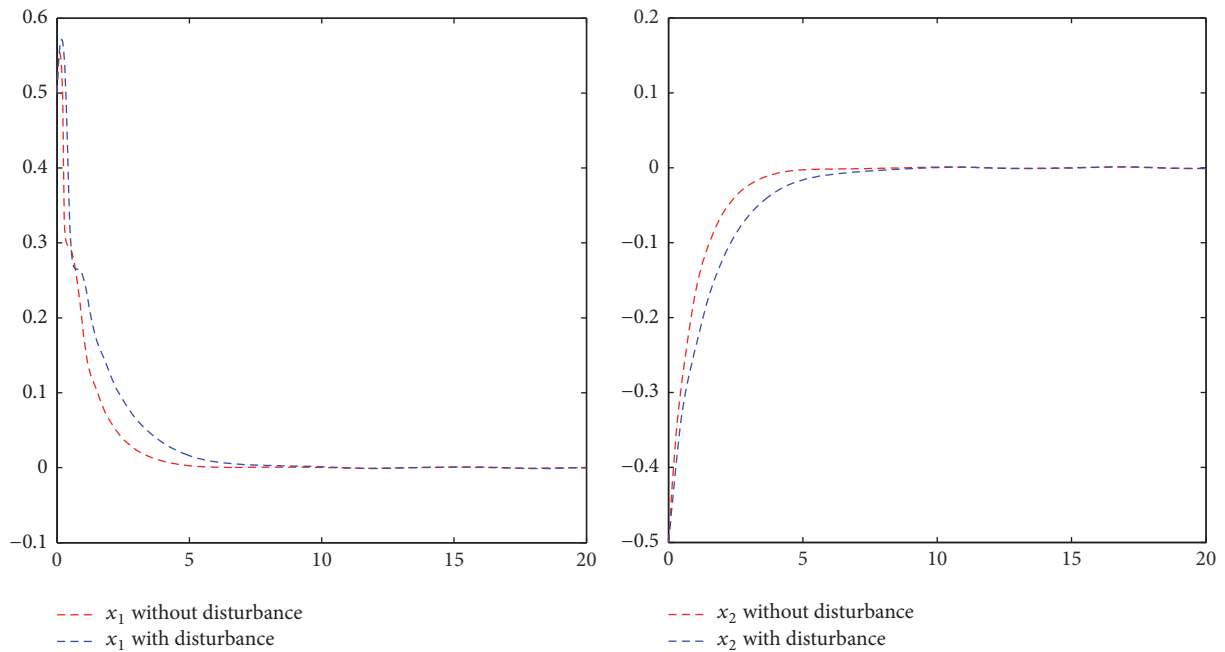


FIGURE 5:  $x_1$  and  $x_2$  with internal disturbance.

$a, b$  are unknown constants, satisfying the fact that  $a_1 \geq a_2 + 1 > 0, a_3 \geq 1, a_4 \geq 1$ .

In stimulation, set  $a_1 = 2, a_2 = 0.5, a_3 = 1.5, a_4 = 1.5, b_1 = 1, b_2 = 2$ .

From Figure 5, to the uncertain system with disturbance, the hybrid bionic adaptive controller proposed makes the system have more anti-interference and good convergence. From Figure 6, when  $C_k = 1$ , the system has good

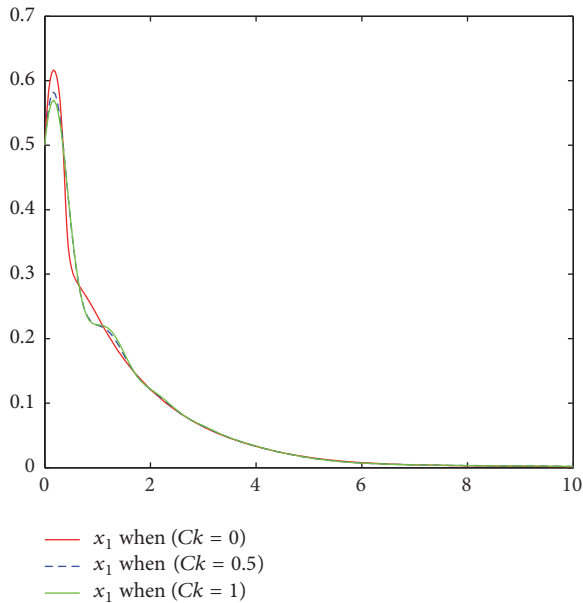


FIGURE 6:  $x_1$  with both internal and external disturbances (when  $C_k$  chooses different values).

convergence and biological adaptability than system controlled by traditional method.

## 7. Conclusion

In this paper, the hybrid fuzzy control method combined model error with tracking error to design the adaptive laws. Compared with traditional method, it has faster convergence rate. In addition, the biological characteristics are embedded in the bionic fuzzy system, which improved the condition that traditional adaptive law just considers the current state of system. The method proposed in this paper makes system have advantages of anti-interference and strong self-adaptability.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

## Authors' Contributions

Yan Li was responsible for the overall revision of the paper and wrote the main manuscript text and put forward some new ideas in the research; Faxiang Zhang provided the dynamic model and the parameter values of a century application; Yimin Li provided the guidance of the paper and was responsible for correspondence.

## Acknowledgments

This research is supported by the National Natural Science Foundation of China: Bionic Intelligent Control Method of Chaotic Ecosystems under Slow Disturbance (11072090). The authors also thank Jing Hua who participated in the

article revision and made a great contribution to the language presentation and revision of first proof.

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